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# Radiation correction to astrophysical fusion reactions and the electron screening problem

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We discuss the effect of electromagnetic environment on laboratory measurements of the nuclear fusion reactions of astrophysical interest. The radiation field is eliminated using the path integral formalism in order to obtain the influence functional, which we evaluate in the semiclassical approximation. We show that enhancement of the tunneling probability due to the radiation correction is extremely small and does not resolve the longstanding problem that the observed electron screening effect is significantly larger than theoretical predictions.

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Nuclear fusion reactions measured at a laboratory at very low incident energies are subjected to the electron screening effect, which originates from bound electrons in the target atom (or molecule). The electrons shield the Coulomb potential between the projectile and the target nuclei, and it is expected that fusion cross sections are enhanced over the prediction which does not take into account this correction, i.e., the fusion cross section calculated for bare nuclei without electrons [1]. Indeed, since the seminal work for the  ${}^3\text{He}(d,p){}^4\text{He}$  reaction [2], the experimental fusion cross sections (or equivalently the astrophysical  $S$  factors) are shown to be enhanced as compared to those extrapolated from the high energy region where the electron screening effect is negligible down to the low energy regime (see Ref. [3] for a recent review).

In the low energy limit, the adiabatic approximation is validated, and the electron screening effect can be expressed as a constant energy shift of the Coulomb potential, where the amount of the shift is given by a difference of electron binding energies between the unified and isolated systems [1]. The experimental data, however, systematically indicate that a significantly larger energy shift is required in order to account for the low energy enhancement of the fusion cross sections [4]. This has been a big surprise and also a rather puzzling situation, since the adiabatic approximation should provide the upper limit of the tunneling probability and thus the upper limit of the energy shift [5–7]. No satisfactory explanation has been proposed to reconcile this problem so far. Furthermore, a recent careful measurement for the stopping power [8,9], as well as recent attempts which used the Trojan-horse method [10] to measure the bare cross sections at low energies [11,12], reconfirmed that the enhancement of fusion cross section is much larger than the model calculation.

Balantekin *et al.* studied several effects beyond the electron screening on astrophysical reactions, which include vacuum polarization, relativity, bremsstrahlung, and atomic polarization [13]. They found that all of these effects are too small to explain the difference between the measured and predicted electron screening energies. Obviously the low energy enhancement of the astrophysical reactions has a differ-

ent origin. Recently, Flambaum and Zelevinsky suggested that the virtual photon emission during tunneling may increase the penetrability [14]. Within a few approximations, which include the closure approximation for the relative motion between the projectile and the target nuclei, they derived a static potential shift due to the radiation correction which is proportional to the Laplace operator applied to the bare internucleus potential. This quantity is exactly zero for the pure Coulomb potential, and the potential renormalization which Flambaum and Zelevinsky derived has a finite value only well inside the Coulomb barrier. Note that such a potential renormalization can be well compensated for with a choice of a nuclear potential between the projectile and the target unless the energy dependence is very strong, as in the case of coupling to high-lying states in heavy-ion subbarrier fusion reactions [15], and it is thus not easy to separate out the radiation effect in a clear way. Since the closure approximation may be too crude for astrophysical reactions, it is necessary to re-examine the effect of virtual photon emission without resorting to the approximations which Flambaum and Zelevinsky used in order to assess the role of radiation field in realistic systems.

The purpose of this paper is to study the effect of coupling between the tunneling motion and the electromagnetic field, taking a different approach from Flambaum and Zelevinsky. To this end, we employ the path-integral formalism for multidimensional tunneling [16] and evaluate the influence functional in the semiclassical approximation. Besides the semiclassical and the dipole approximations, our formalism is exact, and the effect is finite even with the pure Coulomb potential. The path integral approach allows us to discuss both virtual and real photon emissions during tunneling on the equal footing. In this respect, our study is relevant also to the bremsstrahlung in  $\alpha$  decay, which has attracted much interest for the past few years [17–19].

Consider the tunneling motion in the presence of the radiation field. The classical Lagrangian for the system reads [20,21]

$$L = L_0 + L_{\text{em}} + L_{\text{int}}, \quad (1)$$

$$L_0 = \frac{\mu}{2} \dot{\mathbf{r}}^2 - V(r), \quad (2)$$

$$L_{\text{em}} = \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha} \left( \frac{1}{2} \dot{Q}_{k\alpha}^2 - \frac{1}{2} \omega^2 Q_{k\alpha}^2 \right), \quad (3)$$

$$L_{\text{int}} = \sqrt{4\pi} e_{E1} \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha} (\dot{\mathbf{r}} \cdot \boldsymbol{\epsilon}_k^{(\alpha)}) Q_{k\alpha}, \quad (4)$$

where  $\alpha$  represents the polarization index, and  $\boldsymbol{\epsilon}_k^{(\alpha)}$  is the corresponding polarization vector. We employ the Coulomb gauge, and thus the polarization vectors are orthogonal to the photon momentum  $\mathbf{k}$ . We have used the dipole approximation, and introduced the photon coordinate  $Q_{k\alpha}(t)$ , which is independent of the relative motion  $\mathbf{r}$  between the projectile and the target nuclei.  $e_{E1}$  is the  $E1$  effective charge given by  $(A_P Z_T - A_T Z_P)/(A_P + A_T)$ ,  $A_T$  and  $Z_T$  being the mass and the atomic numbers for the target nucleus, respectively, and similar for the projectile also.  $\mu$  is the reduced mass for the relative motion  $\mathbf{r}$ , and  $V(r)$  is the potential between the projectile and the target in the absence of the radiation field. As usual, we have neglected a term which is proportional to  $e_{E1}^2$  in the classical Lagrangian.

Our interest is to compute the transition amplitude for the relative motion from  $\mathbf{r}_i$  to  $\mathbf{r}_f$  while for the radiation field from the vacuum state  $|0\rangle$  to the  $n$ -photon state  $|n\rangle$  for a given incident energy  $E$ . This is expressed in terms of the energy representation of the path integral amplitude as

$$\mathcal{T}_n = g \int_0^\infty dT e^{iET/\hbar} \int \mathcal{D}[\mathbf{r}(t)] e^{(i/\hbar) \int_0^T dt L_0} \times \left\langle n \left| \int \mathcal{D}[Q_{k\alpha}(t)] e^{(i/\hbar) \int_0^T dt (L_{\text{em}} + L_{\text{int}})} \right| 0 \right\rangle, \quad (5)$$

where the kinematical factor  $g$  is proportional to  $\sqrt{E/\mu}$  [16]. Here the integral for  $\mathbf{r}$  is carried out for all paths which connect  $\mathbf{r}(0) = \mathbf{r}_i$  and  $\mathbf{r}(T) = \mathbf{r}_f$ . We square this amplitude and sum over  $n$  in order to obtain the total transition probability for the relative motion,

$$P = \sum_n |\mathcal{T}_n|^2 \quad (6)$$

$$= |g|^2 \int_0^\infty dT e^{iET/\hbar} \int_0^\infty d\tilde{T} e^{-iE\tilde{T}/\hbar} \int \mathcal{D}[\mathbf{r}(t)] \times \int \mathcal{D}[\tilde{\mathbf{r}}(\tilde{t})] e^{(i/\hbar) \int_0^T dt L_0(\mathbf{r})} e^{-(i/\hbar) \int_0^{\tilde{T}} d\tilde{t} L_0(\tilde{\mathbf{r}})} \times \rho[\tilde{\mathbf{r}}(\tilde{t}), \tilde{T}; \mathbf{r}, T], \quad (7)$$

where  $\rho$  is the two-time influence functional given by [16]

$$\rho[\tilde{\mathbf{r}}(\tilde{t}), \tilde{T}; \mathbf{r}, T] = \left\langle 0 \left| \int \mathcal{D}[\tilde{Q}_{k\alpha}(\tilde{t})] e^{-(i/\hbar) \int_0^{\tilde{T}} d\tilde{t} [L_{\text{em}}(\tilde{Q}) + L_{\text{int}}(\tilde{\mathbf{r}}, \tilde{Q})]} \times \int \mathcal{D}[Q_{k\alpha}(t)] e^{(i/\hbar) \int_0^T dt [L_{\text{em}}(Q) + L_{\text{int}}(\mathbf{r}, Q)]} \right| 0 \right\rangle. \quad (8)$$

For the Lagrangian given in Eqs. (1)–(4), the two-time influence functional can be expressed analytically and is given by [16,20,21]

$$\rho[\tilde{\mathbf{r}}(\tilde{t}), \tilde{T}; \mathbf{r}, T] = \exp \left[ \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha} \left\{ -\frac{i}{2} \omega (T - \tilde{T}) - \frac{1}{2\hbar\omega} \times \left( \int_0^T dt \int_0^{\tilde{T}} ds f_{k\alpha}(t) f_{k\alpha}(s) e^{-i\omega(t-s)} + \int_0^{\tilde{T}} dt \int_0^T ds \tilde{f}_{k\alpha}(t) \tilde{f}_{k\alpha}(s) e^{i\omega(t-s)} - e^{-i\omega(T-\tilde{T})} \times \int_0^T dt f_{k\alpha}(t) e^{i\omega t} \int_0^{\tilde{T}} ds \tilde{f}_{k\alpha}(s) e^{-i\omega s} \right) \right\} \right] \quad (9)$$

where  $f_{k\alpha}$  and  $\tilde{f}_{k\alpha}$  are defined by

$$f_{k\alpha}(t) \equiv -\sqrt{4\pi} e_{E1} \dot{\mathbf{r}} \cdot \boldsymbol{\epsilon}_k^{(\alpha)}, \quad \tilde{f}_{k\alpha}(t) \equiv -\sqrt{4\pi} e_{E1} \dot{\tilde{\mathbf{r}}} \cdot \boldsymbol{\epsilon}_k^{(\alpha)}. \quad (10)$$

In the exponent in the influence functional [Eq. (9)], the last term is the real photon contribution, while the second and the third terms represent the virtual photon emission [20].

In the radiation problems such as the Lamb shift calculation, it is essential to separate out a divergent contribution due to the mass renormalization in order to obtain a physical result. For our problem, this can be done by performing a partial integration for the second and the third terms in Eq. (9):

$$\begin{aligned} & \int_0^T dt \int_0^{\tilde{T}} ds f_{k\alpha}(t) f_{k\alpha}(s) e^{-i\omega(t-s)} \\ &= \frac{1}{i\omega} \int_0^T dt [f_{k\alpha}(t)]^2 - \frac{1}{i\omega} \int_0^T dt f_{k\alpha}(t) f_{k\alpha}(0) e^{-i\omega t} \\ & \quad - \frac{1}{i\omega} \int_0^T dt \int_0^{\tilde{T}} ds f_{k\alpha}(t) \frac{df_{k\alpha}(s)}{ds} e^{-i\omega(t-s)}. \end{aligned} \quad (11)$$

The first term is proportional to  $\dot{\mathbf{r}}^2$ , and is nothing but the mass renormalization, which has already included in the kinetic term in Eq. (2). The second term vanishes if we choose  $t=0$  at the outer classical turning point (note that we are going to deal with the tunneling problem in the following). Retaining only the third term, the regularized influence functional  $\rho_{\text{reg}}$  then reads

$$\begin{aligned} \rho_{\text{reg}}(\tilde{\mathbf{r}}(\tilde{t}), \tilde{T}; \mathbf{r}, T) &= \exp \left[ \int \frac{d^3 k}{(2\pi)^3} \sum_{\alpha} \left( -\frac{1}{2\hbar\omega} \right) \right. \\ &\times \left\{ -\frac{1}{i\omega} \int_0^T dt \int_0^t ds f_{k\alpha}(t) \frac{df_{k\alpha}(s)}{ds} e^{-i\omega(t-s)} \right. \\ &+ \frac{1}{i\omega} \int_0^{\tilde{T}} dt \int_0^t ds \tilde{f}_{k\alpha}(t) \frac{d\tilde{f}_{k\alpha}(s)}{ds} e^{i\omega(t-s)} \\ &\left. \left. - e^{-i\omega(T-\tilde{T})} \int_0^T dt f_{k\alpha}(t) e^{i\omega t} \int_0^{\tilde{T}} ds \tilde{f}_{k\alpha}(s) e^{-i\omega s} \right\} \right]. \end{aligned} \quad (12)$$

Here we have also dropped the  $-(i/2)\omega(T-\tilde{T})$  term, since it merely changes the definition of the energy of the vacuum state.

We now introduce the semiclassical approximation to the path integrals with respect to  $\mathbf{r}$  as well as to the time integrals in Eq. (7) [6,22,23]. This leads to a classical tunneling path from the outer to the inner turning points along the imaginary time axis. For simplicity, in the following, we consider only a head-on collision. Since the coupling strength  $e^2_{E1}/\hbar c$  is small, we determine the tunneling path by disregarding the radiation field. The classical path thus obeys the Newtonian equation with the inverted potential,

$$\mu \frac{d^2 z_{\text{cl}}}{d\tau^2} = \frac{d}{dz} V(z_{\text{cl}}). \quad (13)$$

The influence functional is then evaluated along the classical path. After carrying out the angle integral for the photon momentum  $\mathbf{k}$ , penetrability (7) in the semiclassical approximation thus reads

$$P = P_0 f, \quad (14)$$

where

$$\begin{aligned} P_0 &= |g|^2 \int_0^\infty dT e^{iET/\hbar} \int_0^\infty d\tilde{T} e^{-iE\tilde{T}/\hbar} \\ &\times \int \mathcal{D}[z(t)] \int \mathcal{D}[\tilde{z}(\tilde{t})] e^{(i/\hbar) \int_0^T dt L_0(z)} e^{-(i/\hbar) \int_0^{\tilde{T}} d\tilde{t} L_0(\tilde{z})}, \end{aligned} \quad (15)$$

is the penetrability in the absence of the radiation field, and

$$f = \rho_{\text{reg}}(z_{\text{cl}}(t)^*, T_{\text{cl}}^*; z_{\text{cl}}(t), T_{\text{cl}}) \quad (16)$$

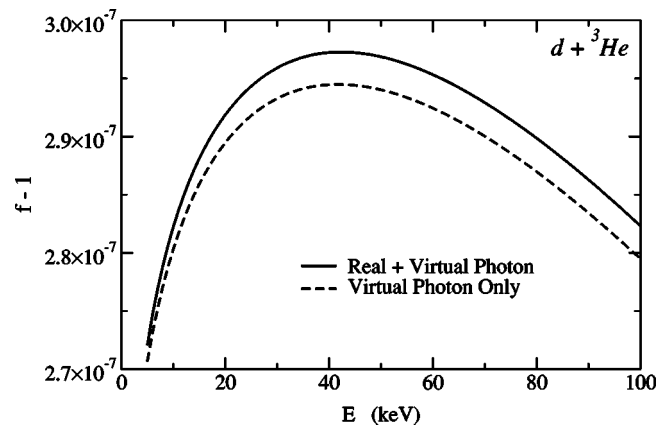


FIG. 1. The deviation of the enhancement factor  $f$  due to the radiation coupling from unity as a function of the incident energy for the  $d + {}^3\text{He}$  fusion reaction. The solid line includes both the real and virtual photon emissions during tunneling, whereas the dashed line represents only the virtual photon contribution.

$$\begin{aligned} &= \exp \left[ \int_0^\infty \frac{k^2 dk}{(2\pi)^3} \frac{1}{2\hbar\omega} \right. \\ &\times \frac{8\pi}{3} \left\{ \frac{2}{\omega} \int_0^{T_0} d\tau \int_0^\tau d\tau' 4\pi e^2_{E1} \frac{1}{\mu} \frac{dz}{d\tau} \frac{dV}{dz(\tau')} e^{-\omega(\tau-\tau')} \right. \\ &\left. \left. + 4\pi e^2_{E1} e^{-2\omega T_0} \left( \int_0^{T_0} d\tau \frac{dz}{d\tau} e^{\omega\tau} \right)^2 \right\} \right] \end{aligned} \quad (17)$$

is the regularized two-time influence functional along the classical trajectory  $z_{\text{cl}}$ . In obtaining  $f$ , we have introduced the imaginary time evolutions  $t \rightarrow -i\tau, s \rightarrow -i\tau'$ , and  $T \rightarrow -iT_0$ , and neglected the excitation of the radiation field prior to the tunneling.  $T_0$  is the Euclidean tunneling time, where  $z_{\text{cl}}(T_0)$  corresponds to the inner turning point  $R_0$  while  $z_{\text{cl}}(0)$  is the outer turning point  $Z_P Z_T e^2/2E$ .

Let us now numerically estimate the enhancement factor  $f = P/P_0$  for the  $d + {}^3\text{He}$  reaction. To this end, we consider the pure Coulomb potential  $V(r) = Z_P Z_T e^2/r$  from the outer turning point  $Z_P Z_T e^2/E$  to the inner turning point  $R_0 = 4.3$  fm [13]. Newtonian equation (13) can then be solved analytically as

$$\tau = \frac{Z_P Z_T e^2}{2E} \sqrt{\frac{\mu}{2E}} (w + \sin w), \quad z(\tau) = \frac{Z_P Z_T e^2}{2E} (1 + \cos w). \quad (18)$$

Because of the exponential factor, the  $k$  integral in Eq. (17) is quickly damped as a function of  $k$ , and it is not necessary to introduce the momentum cutoff factor for our tunneling problem. Figure 1 shows the deviation of the enhancement factor  $f$  from unity as a function of the incident energy. We see that the enhancement due to the radiation coupling in the tunneling region is extremely small, and does not play any important role in the astrophysical fusion reaction. The dashed line shows the virtual photon contribution separately, which is obtained by neglecting the second term in Eq. (17). One finds that the virtual photon emission provides the domi-

TABLE I. The enhancement factor  $f$  for several reactions obtained at the lowest experimental energy  $E_{\min}$  and with the inner turning point  $R_0$  indicated.

Reaction	$E_{\min}$ (keV)	$R_0$ (fm)	$f-1$
$^3\text{He}(d,p)^4\text{He}$	5.88	4.3	$2.744 \times 10^{-7}$
$\text{D}(^3\text{He},p)^4\text{He}$	5.38	4.3	$2.731 \times 10^{-7}$
$^6\text{Li}(p,\alpha)^3\text{He}$	10.74	3.0	$3.758 \times 10^{-6}$
$^7\text{Li}(p,\alpha)^4\text{He}$	12.70	4.3	$3.598 \times 10^{-6}$
$\text{H}(^6\text{Li},\alpha)^3\text{He}$	10.94	3.0	$3.761 \times 10^{-6}$
$\text{H}(^7\text{Li},\alpha)^4\text{He}$	12.97	4.3	$3.602 \times 10^{-6}$
$^{10}\text{B}(p,\alpha)^7\text{Be}$	18.70	3.3	$6.095 \times 10^{-6}$
$^{11}\text{B}(p,\alpha)^8\text{Be}$	16.70	2.0	$1.169 \times 10^{-5}$

nant contribution, although the real photon emission is not negligible. Note that even the real photon emission may enhance the penetrability. This is due to a dynamical modulation of potential between the projectile and the target caused by the radiation coupling. The coupling assisted tunneling has been well known in heavy-ion fusion reactions at sub-barrier energies [16,24,25]. Although we determine the classical path without taking into account the energy loss due to the real photon emission, the effect of the dynamical modulation of the potential is much larger than the perturbation to the classical path, and the penetrability will remain enhanced due to the real photon emission even after the classical path is determined self-consistently as in Refs. [22,23]. Table I shows the enhancement factor for several reactions. We take the same value for the inner turning point  $R_0$  as in Ref. [13]. In all the reactions considered, the effects of the radiation

coupling during tunneling is almost negligible and the enhancement factor never exceeds  $10^{-3}\%$ . This effect is even much smaller than the other small effects considered in Ref. [13]. We thus conclude that the radiation correction is not helpful for the low energy enhancement of nuclear astrophysical reactions.

In summary, we have studied the effect of the radiation coupling on the tunneling motion of charged particles. To this end, we have employed the semiclassical approximation to the transition amplitude in the path integral representation. We have shown that the effect is almost negligible for nuclear astrophysical reactions, and the large electron screening problem remains unsolved. The radiation correction which we discussed in this paper is even smaller than the vacuum polarization effect, and would be negligible as compared to the latter. In the present paper, we discussed only the radiation correction to the nuclear fusion process without the electrons. In the adiabatic approximation, the electrons contribute only as a constant shift in the potential energy between the projectile and the target, and therefore our result will not be altered qualitatively even after the electrons are taken into account. The origin of the discrepancy between the measured and the calculated electron screening energies still seems to be an open problem.

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